

Math 221-Midterm 1 Practice exam 1

1. (13 points)

- (a) (6 points) Use **the definition of the derivative as a limit** to find the derivative of $f(x) = x + \frac{1}{x}$ at $x = 1$.

- (b) (7 points) Use **the definition of the derivative as a limit** to find the equation of the tangent line to the curve $y = \sqrt{x+1}$ at $x = 3$.

2. (20 points) Calculate the derivatives of the following functions by using differentiation rules.

(a) $y = (10x + 4)^{2012}$

(b) $y = x \sin x \cos x$

(c) $y = \frac{x^2}{\sin(\cos x)}$

(d) $y = \frac{\sin x}{1 + \cos x}$ (Please simplify your answer as much as possible)

3. (20 points) Compute the following limits or show that it does not exist. If you think the limit does not exist, explain why, and specify if the limit is ∞ or $-\infty$.

(a) $\lim_{x \rightarrow -2} \frac{|x + 2|}{x(x + 2)}.$

(b) $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2}$ (Hint: Use the double-angle formula $\cos(2x) = 1 - 2\sin^2(x)$).

(c) $\lim_{x \rightarrow \infty} \frac{\sin(2x)}{x}$

(d) $\lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{(\pi - 2x)^2}$

4. (10 points) For each of the problems, circle the (unique) correct answer.

(a) Only one of the following limits exists. Which one?

A. $\lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right)$

B. $\lim_{x \rightarrow \infty} \sin x$

C. $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$

D. $\lim_{x \rightarrow 0} \frac{1}{\sin x}$

(b) $\lim_{x \rightarrow -1-} \frac{x}{x^2 - x - 2}$ is

A. ∞

B. $-\infty$

C. 0.

D. None of the above.

(c) Suppose $f(x)$ is an odd function and $\lim_{x \rightarrow 1+} f(x) = 2$. What can we say about $\lim_{x \rightarrow -1+} f(x)$?

A. $\lim_{x \rightarrow -1+} f(x) = 2$

B. $\lim_{x \rightarrow -1+} f(x) = -2$

C. *There is not enough information to tell what $\lim_{x \rightarrow -1+} f(x)$ is.*

(d) If $f(x)$ and $g(x)$ are functions defined near $x = 1$ and $\lim_{x \rightarrow 1} f(x) = 0$, then $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)} = 0$

A. True

B. False

(e) If $f'(0)$ exists, then $\lim_{x \rightarrow 0} f(x) = f(0)$.

A. True

B. False

5. (21 points) Do the following problems about finding asymptotes.

(a) (7 points) Find all the horizontal asymptotes of $f(x) = \frac{2x^2 + 2x}{x^2 - x - 2}$. Show your work.

(b) (7 points) Find all the vertical asymptotes of $f(x) = \frac{2x^2 + 2x}{x^2 - x - 2}$. Show your work.

(c) (7 points) Find the slanted asymptote of $g(x) = \frac{3x^2 + x}{x + 1}$ as $x \rightarrow \infty$. Show your work.

6. (16 points) Consider the function $f(x)$ defined by $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ k(x-1) + 1 & \text{if } x > 1 \end{cases}$ where k is a constant and answer the following questions.

(a) For what value (or values) of k is $f(x)$ continuous at $x = 1$? Justify your answer.

(b) For what value (or values) of k is $f(x)$ differentiable at $x = 1$? Justify your answer.